



Oxford Cambridge and RSA

**Friday 17 June 2022 – Afternoon**

**AS Level Further Mathematics A**

**Y535/01 Additional Pure Mathematics**

**Time allowed: 1 hour 15 minutes**



**You must have:**

- the Printed Answer Booklet
- the Formulae Booklet for AS Level Further Mathematics A
- a scientific or graphical calculator

**INSTRUCTIONS**

- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to **3** significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by  $g \text{ m s}^{-2}$ . When a numerical value is needed use  $g = 9.8$  unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

**INFORMATION**

- The total mark for this paper is **60**.
- The marks for each question are shown in brackets [ ].
- This document has **8** pages.

**ADVICE**

- Read each question carefully before you start your answer.

Answer **all** the questions.

- 1 The points  $A$ ,  $B$  and  $C$  have position vectors  $\mathbf{a} = \begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix}$ ,  $\mathbf{b} = \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix}$  and  $\mathbf{c} = \begin{pmatrix} -5 \\ 1 \\ 2 \end{pmatrix}$  respectively, relative to the origin  $O$ .
- (a) Calculate, in its simplest exact form, the area of triangle  $OAB$ . [3]
- (b) Show that  $\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) + \mathbf{b} \times (\mathbf{c} \times \mathbf{a}) + \mathbf{c} \times (\mathbf{a} \times \mathbf{b}) = \mathbf{0}$ . [3]
- 2 The surface  $S$  has equation  $z = x^3 + y^3 - 2x^2 - 5y^2 + 3xy$ .
- It is given that  $S$  has two stationary points; one at the origin,  $O$ , and the other at the point  $A$ .
- Determine the coordinates of  $A$ . [6]
- 3 The sequence  $\{U_n\}$  is given by  $U_1 = 0$ ,  $U_2 = -1$  and  $U_{n+2} = U_{n+1} + U_n + n - 1$  for  $n \geq 1$ .
- (a) List the first seven terms of this sequence. [2]
- The Fibonacci sequence  $\{F_n\}$  is given by  $F_1 = 1$ ,  $F_2 = 1$  and  $F_{n+2} = F_{n+1} + F_n$  for  $n \geq 1$ .
- (b) (i) By comparing the two sequences, give the relationship between  $U_n$  and  $F_n$ . [2]
- (ii) Show that the relationship found in part (b)(i) holds for all  $n \geq 1$ . [3]
- 4 Let  $N = 10a + b$  and  $M = a + 3b$ , where  $a$  and  $b$  are integers such that  $a \geq 1$  and  $0 \leq b \leq 9$ .
- (a) Prove that  $29 \mid N$  if and only if  $29 \mid M$ . [5]
- (b) Use an iterative method based on the result of part (a) to show that  $899\,364\,472$  is a multiple of 29. [3]

- 5 A research student is using 3-D graph-plotting software to model a chain of volcanic islands in the Pacific Ocean. These islands appear above sea-level at regular intervals, (approximately) distributed along a straight line. Each island takes the form of a single peak; also, along the line of islands, the heights of these peaks decrease in size in an (approximately) regular fashion (see Fig. 1.1).

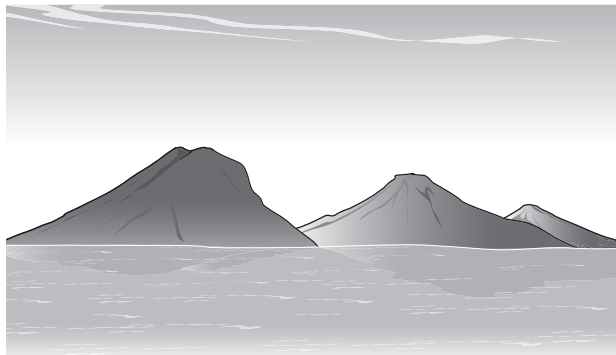


Fig. 1.1

The student's model uses the surface with equation  $z = \sin x + \sin y$ , a part of which is shown in Fig. 1.2 below. The surface of the sea is taken to be the plane  $z = 0$ .

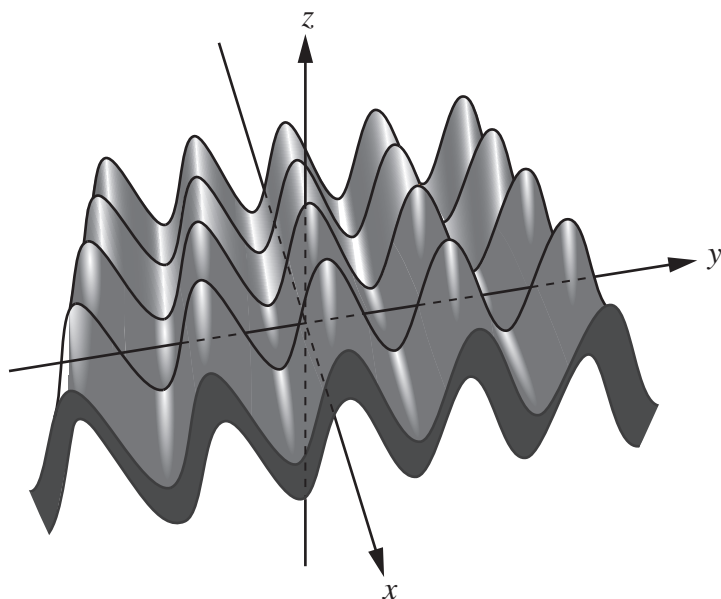


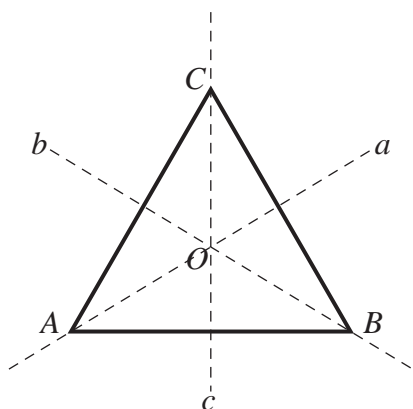
Fig. 1.2

- (a) • Describe **two** problems with this model.  
 • Suggest revisions to this model so that each of these problems is addressed. [4]
- (b) Still using their original model, the student examines the contour  $z = 2$  for their surface only to find that the software shows what appears to be an empty graph.

Explain what has happened.

[2]

- 6 The sequence  $\{u_n\}$  is such that  $u_1 = 7, u_2 = 37, u_3 = 337, u_4 = 3337, \dots$ .
- (a) Write down a first-order recurrence system for  $\{u_n\}$ . [2]
- (b) By solving the recurrence system of part (a), show that  $u_n = \frac{1}{3}(10^n + 11)$ . [5]
- (c) Prove that  $\{u_n\}$  contains infinitely many terms which are multiples of 37. [7]
- 7 The diagram below shows an equilateral triangle  $ABC$ . The three lines of reflection symmetry of  $ABC$  (the lines  $a, b$  and  $c$ ) are shown as broken lines. The point of intersection of these three lines,  $O$ , is the centre of rotational symmetry of the triangle.



The group  $D_3$  is defined as the set of symmetries of  $ABC$  under the composition of the following transformations.

$i$ : the identity transformation

$a$ : reflection in line  $a$

$b$ : reflection in line  $b$

$c$ : reflection in line  $c$

$p$ : an anticlockwise rotation about  $O$  through  $120^\circ$

$q$ : a clockwise rotation about  $O$  through  $120^\circ$

Note that the lines  $a, b$  and  $c$  are unaffected by the transformations and remain fixed.

- (a) On the diagrams provided in the Printed Answer Booklet, show each of the six elements of  $D_3$  obtained when the above transformations are applied to triangle  $ABC$ . [3]
- (b) Complete the Cayley table given in the Printed Answer Booklet. [3]

- (c) List all the proper subgroups of  $D_3$ . [3]
- (d) State, with justification, whether  $D_3$  is
- (i) cyclic, [1]
  - (ii) abelian. [1]
- (e) The group  $H$ , also of order 6, is the set of rotational symmetries of the regular hexagon. Describe **two structural** differences between  $D_3$  and  $H$ . [2]

**END OF QUESTION PAPER**

**BLANK PAGE**

**BLANK PAGE**

---

# OCR

Oxford Cambridge and RSA

## Copyright Information

OCR is committed to seeking permission to reproduce all third-party content that it uses in its assessment materials. OCR has attempted to identify and contact all copyright holders whose work is used in this paper. To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced in the OCR Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download from our public website ([www.ocr.org.uk](http://www.ocr.org.uk)) after the live examination series. If OCR has unwittingly failed to correctly acknowledge or clear any third-party content in this assessment material, OCR will be happy to correct its mistake at the earliest possible opportunity.

For queries or further information please contact The OCR Copyright Team, The Triangle Building, Shaftesbury Road, Cambridge CB2 8EA.

OCR is part of Cambridge University Press & Assessment, which is itself a department of the University of Cambridge.